OCR
Oxford Cambridge and RSA

Monday 27 June 2016 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

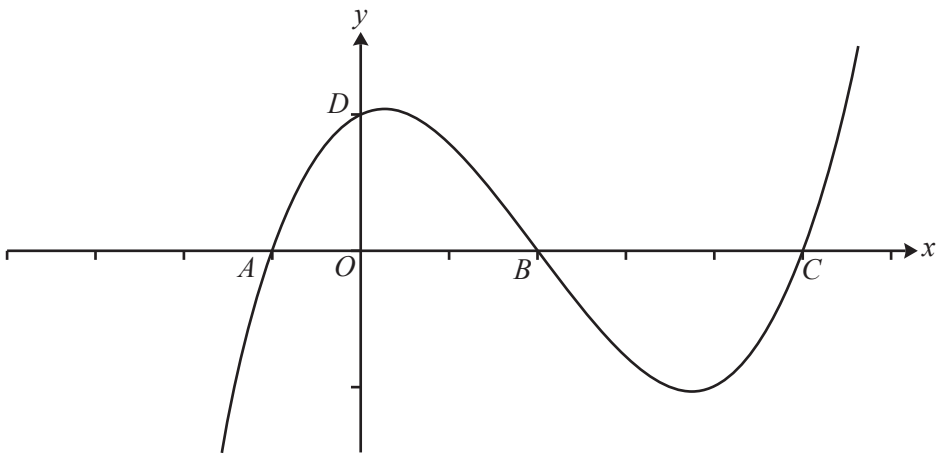
Answer **all** the questions.

1 **(i)** By first expanding $(e^x + e^{-x})^3$, or otherwise, show that $\cosh 3x \equiv 4\cosh^3 x - 3\cosh x$. **[4]**

(ii) Solve the equation $\cosh 3x = 6\cosh x$, giving your answers in exact logarithmic form. **[5]**

2 It is given that $f(x) = \dfrac{x(x-1)}{(x+1)(x^2+1)}$. Express $f(x)$ in partial fractions and hence find the exact value of $\displaystyle\int_0^1 f(x)\,dx$. **[6]**

3 The diagram shows the curve $y = f(x)$. Points A, B, C and D on the curve have coordinates (–1, 0), (2, 0), (5, 0) and (0, 2) respectively.



On the copy of this diagram in the Printed Answer Book, sketch the curve $y^2 = f(x)$, giving the coordinates of the points where the curve crosses the axes. **[5]**

4 You are given the equation $(2x-1)^2 - e^x = 0$.

(i) Verify that 0 is a root of the equation. **[1]**

There are also two other roots, α and β, where $0 < \alpha < \beta$.

(ii) The iterative formula $x_{r+1} = \ln(2x_r - 1)^2$ is to be used to find a root of the equation.

(a) Sketch the line $y = x$ and the curve $y = \ln(2x-1)^2$ on the same axes, showing the roots 0, α and β. **[3]**

(b) By drawing a 'staircase' diagram on your sketch, starting with a value of x that is between α and β, show that this iteration does not converge to α. **[1]**

(c) Using this iterative formula with $x_1 = 3.75$, find the value of β correct to 3 decimal places. **[3]**

(iii) Using the Newton-Raphson method with $x_1 = 1.6$, find the root α of the equation $(2x-1)^2 - e^x = 0$ correct to 5 significant figures. Show the result of each iteration. **[4]**

5 It is given that $y = \tan^{-1} 2x$.

(i) Find $\frac{dy}{dx}$ and show that $\frac{d^2y}{dx^2} + 4x\left(\frac{dy}{dx}\right)^2 = 0$. **[3]**

(ii) Find the Maclaurin series for y up to and including the term in x^3. Show all your working. **[4]**

(iii) The result in part **(ii)**, together with the value $x = \frac{1}{2}$, is used to find an estimate for π. Show that this estimate is only correct to 1 significant figure. **[2]**

6 The equation of a curve in polar coordinates is $r = \sin 5\theta$ for $0 \leqslant \theta \leqslant \frac{1}{5}\pi$.

(i) Sketch the curve and write down the equations of the tangents at the pole. **[4]**

(ii) The line of symmetry meets the curve at the pole and at one other point A. Find the equation of the line of symmetry and the cartesian coordinates of A. **[2]**

(iii) Find the area of the region enclosed by this curve. **[4]**

7 **(i)** By using a set of rectangles of unit width to approximate an area under the curve $y = \frac{1}{x}$, show that $\sum_{x=1}^{\infty} \frac{1}{x}$ is infinite. **[4]**

(ii) By using a set of rectangles of unit width to approximate an area under the curve $y = \frac{1}{x^2}$, find an upper limit for the series $\sum_{x=1}^{\infty} \frac{1}{x^2}$. **[5]**

8 It is given that $I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \, \mathrm{d}x$ where n is a positive integer.

(i) By writing $\sec^n x = \sec^{n-2} x \sec^2 x$, or otherwise, show that

$$(n-1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2} \text{ for } n > 1.$$ **[5]**

(ii) Show that $I_8 = \frac{96}{35}$. **[3]**

(iii) Prove by induction that I_{2n} is rational for all values of $n > 1$. **[4]**

END OF QUESTION PAPER